

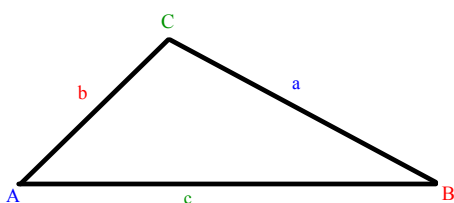
In non-right triangles we cannot use the primary trigonometric ratio; there is no  $90^\circ$  angle, so there is no hypotenuse!

However, there still exists relationships between the sides and the angles in the triangle.

The relationships can be expressed in terms of sine or cosine and are called the Sine Law and the Cosine Law.

We will study these laws over the next few days.

### L8(8.4) - The Cosine Law



The Cosine Law (2 formats) for  $\triangle ABC$ :

$$a^2 = b^2 + c^2 - 2bc \cos A$$

or

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

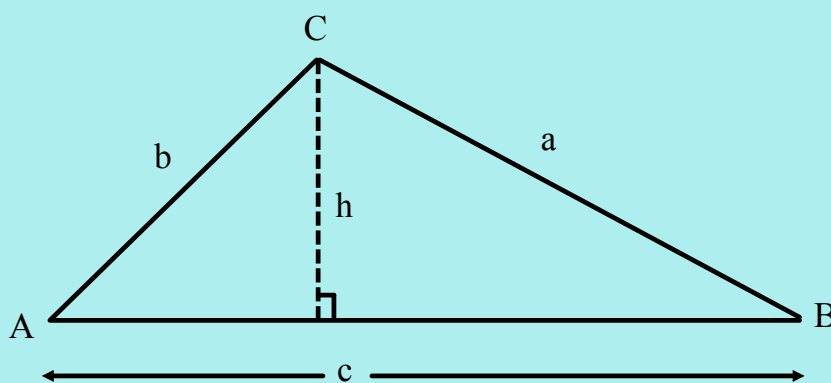
You decide which format to use depending on what you are solving for.

The Cosine Law can be re-written for the other sides and angles of the triangle:

$$b^2 = a^2 + c^2 - 2ac \cos B \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

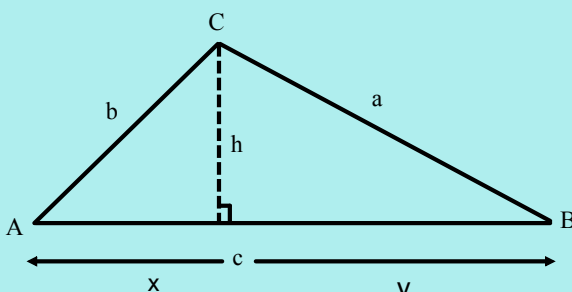
$$c^2 = a^2 + b^2 - 2ab \cos C \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Proving the Cosine Law:



We can always create right triangles by drawing an altitude from any vertex.

Using trigonometry on each right triangle, we can relate the angles and sides of the overall triangle.



we will need:

$$x^2 + h^2 = b^2 \quad y^2 + h^2 = a^2$$

$$h^2 = b^2 - x^2 \quad h^2 = a^2 - y^2$$

set  $h^2 = h^2$

$$a^2 - y^2 = b^2 - x^2$$

$$a^2 = b^2 - x^2 + y^2$$

sub.  $y = c - x$

$$a^2 = b^2 - x^2 + (c - x)^2$$

$$a^2 = b^2 - x^2 + c^2 - 2cx + x^2$$

$$a^2 = b^2 + c^2 - 2cx$$

sub.  $x = b \cos A$

$$a^2 = b^2 + c^2 - 2cb \cos A$$

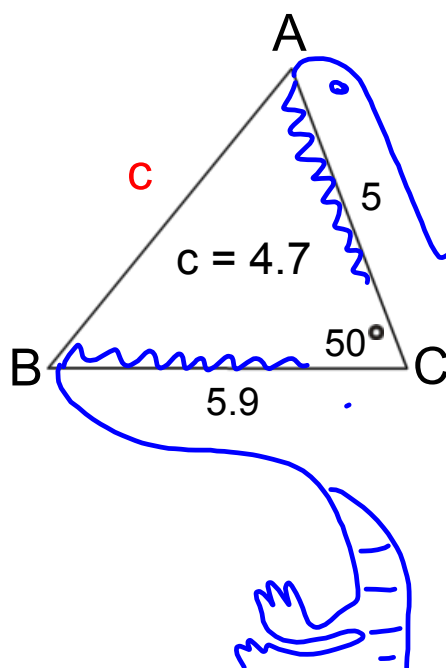
Also we need:

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos A = \frac{x}{b}$$

$$b \cos A = x$$

Ex. 1 Find the length of side c.

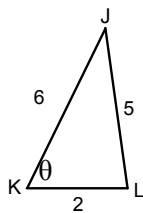


$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 5.9^2 + 5^2 - 2(5.9)(5) \cos 50^\circ$$

$$c^2 = 21.885$$

$$c = 4.68$$

Ex.2 Solve for  $\theta$ .

$$\cos K = \frac{l^2 + j^2 - k^2}{2lj}$$

$$\cos K = \frac{6^2 + 2^2 - 5^2}{2(6)(2)}$$

$$\cos K = \frac{15}{24}$$

$$\cos^{-1}(\cos K) = \cos^{-1}\left(\frac{15}{24}\right)$$

$$K = 51.32$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$5^2 = 6^2 + 2^2 - 2(2)(6) \cos C$$

$$25 = 36 + 4 - 24 \cos C$$

$$24 \cos C = 36 + 4 - 25$$

$$\frac{24 \cos C}{24} = \frac{15}{24}$$

$$\cos C = \frac{15}{24}$$

Assigned Work:

p.438 # 2, 3b

p.443 # 2, 3, 4, 5ac